

Name: ..... **I.D** .....

***In computing limits, you may skip the following***

*1) certain justifications if they are well-known.*

*2) In LCT, IF you are sure that  $L=1$ , write  $L=1$  & skip its proof.*

**Investigate** = Investigate for convergence or divergence

1. (a) Investigate the series  $\sum_{n=1}^{\infty} \left(\frac{en+1}{en+7}\right)^n$

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
<b>Total over 100</b>	

(b) Investigate  $\sum_{n=1}^{\infty} a_n$  given that  $\left|\frac{a_{n+1}}{a_n}\right| = \frac{7n^2}{(3n+2)(2n+1)} - \frac{\cos(n+5)}{n}$

(c) Investigate  $\sum_{n=1}^{\infty} a_n$  given that  $\left| \frac{a_{n+1}}{a_n} \right| = 1 + \frac{n!}{n^n}$

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d) Investigate  $\sum_{n=1}^{\infty} \sin\left(\frac{7^n}{n!}\right)$

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e) Investigate  $\sum_2^{\infty} \ln\left(1 + \frac{3}{n \ln n}\right)$

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f) Investigate  $\sum_{n=1}^{\infty} (\sqrt[n]{n} - 1)^{3/2}$  (BIG Hint:  $\sqrt[n]{n} = e^{\frac{\ln n}{n}}$ )

2a) Investigate  $\sum_1^{\infty} (-1)^n \frac{100^n (n!)^3}{(3n)!}$

2a') Find  $\lim_{n \rightarrow \infty} \frac{n^3}{\sqrt[3]{(3n)!}}$  **Hint:** Use  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$  (if this last exists) .

2b) Find the domain of convergence **and the domain of Absolute convergence** of

$$\sum_{n=2}^{\infty} (-1)^n \frac{(2x-3)^n}{7^n n \ln n}$$

**3a)** Find the Maclaurin series of  $f(x) = e^{-5x^3}$  to deduce  $f^{(150)}(0)$

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**3b) Estimate**  $\int_0^{0.2} e^{-5x^3} dx$  by using the first 3 terms of the appropriate series & find **|Error|** < ....

4a) (**Suppose**  $\sum_{n=1}^{\infty} a_n$  **converges**, prove or disprove that  $\sum_{n=1}^{\infty} \ln(2 + \sin(a_n))$  converges

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4b) **Suppose**  $\sum_{n=1}^{\infty} a_n$  **converges**, prove or disprove that  $\sum_{n=1}^{\infty} (-1)^n \left(\frac{5 + \sin(a_n)}{4}\right)^n$  converges

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4c) Investigate carefully  $\sum_{n=1}^{\infty} (-1)^n \frac{\sin(n)}{n\sqrt{n}}$

5a) Find  $\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}}}{5\sqrt{n}}$

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5b) Use Taylor's Remainder's formula to show that  $f(x) =$  its Maclaurin series given that  $|f^{(n)}(x)| \leq (1000)^{7n} e^x$  for all  $n$  & for all  $x$ . (For time limitations, assume  $x > 0$ )

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5B) Use Taylor's Remainder formula to find the maximum possible Error (as a simple fraction)

in the approximation of  $\sum_{n=0}^{\infty} \frac{(0.7)^n}{n!}$  using only the first 3 terms.

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5c) Use the Maclaurin series of  $e^x$  to prove  $\sqrt[n]{n!} > \frac{n}{e}$  for all  $n$ .